St George's School
Further Mathematics
KS5 Curriculum

PREREQUISITE KNOWLEDGE & SKILLS *The foundations needed to thrive in this subject.*	**Who should study this subject?** Students who are studying A Level Mathematics who have a real flair and passion for the subject and wish to study it in greater breadth and depth. Studying Further Mathematics is particularly beneficial for those who wish to read Mathematics and/or Engineering at university. **Key Skills developed during KS4:** As per A Level Mathematics **St George's course entry requirements:** In addition to the entry requirement for sixth form, a **grade 7** or above in Mathematics. Please note that Mathematics and Further Mathematics can only be taken **together** with **2 other** A Level courses.
QUALIFICATION *Exam Board, aims and objectives.*	**A Level Further Mathematics OCR H245** https://www.ocr.org.uk/qualifications/as-and-a-level/further-mathematics-a-h235-h245-from-2017/
ASSESSMENT *Internal monitoring and final assessment.*	**Internal Assessment:** • Pure Core Mathematics: Internal checkpoints are taken at regular intervals, 2 or 3 topics at a time. • Discrete/Mechanics/Statistics: Internal checkpoints are taken when the AS Level material has been completed. • January Year 13: Mock examinations **Final assessment:** 4 x 90 minute examinations taken at the end of the course: • Mandatory Pure Core (50%): Pure Core 1 & Pure Core 2 • Option 1 (25%): Discrete Mathematics • Option 2 (25%): Mechanics **or** Statistics
ENRICHMENT *Trips & Visits, wider reading, etc.*	**Visits and Events:** • Senior Maths Challenge • Maths Team Challenge • University of Hertfordshire Problem Solving Workshops **Wider reading:** • Books by Marcus du Sautoy, Rob Eastaway, Hannah Fry, Simon Singh and Ian Stewart • Podcasts including More or Less, Infinite Monkey Cage, Curious Cases of Rutherford and Fry There are various websites which are useful for wider reading: • http://www.cut-the-rope.org/ • https://plus.maths.org/content/
NEXT STEPS *Where this subject can take you.*	**Related University Courses:** As per A Level Mathematics. In addition, students following degree courses in Mathematics and/or Engineering would find Further Mathematics particularly useful. **Career Paths:** As per A Level Mathematics.

Students complete the A Level Mathematics course in one year, gaining the qualification at the end of Year 12. Please see A Level Mathematics for course content. Students then start the Further Mathematics course at the end of Year 12.

	Year 12
Autumn Term	Students will follow the Year 12 Mathematics course
Spring Term	Students will follow the Year 13 Mathematics course
Summer Term	**Topics:** **Pure Core** Matrices and their Applications; Complex Numbers; and Further Vectors **Skills:** Adding, subtracting and performing scalar multiplication with conformable matrices Learning about zero and identity matrices and their significance Calculating the determinant of a 2 x 2 or 3 x 3 matrix Finding and interpreting the inverse of a 2 x 2 or 3 x 3 matrix, where one exists Writing an equation of a straight line in 3 dimensions, using both vectors and coordinates Finding the intersection point of 2 lines Calculating an angle between 2 vectors or 2 straight lines (using a scalar product) Deciding whether 2 lines are parallel or perpendicular Finding a line that is perpendicular to 2 given lines (using a vector product) Solving problems involving distances between points and lines Using matrices to solve sets of simultaneous equations Interpreting matrices as linear transformations in 2 and 3 dimensions Finding a matrix representing a combined transformation Finding invariant points and invariant lines of a linear transformation Learning about a new set of numbers called the complex numbers Performing arithmetic with complex numbers Understanding why complex roots of real polynomials occur in conjugate pairs Representing complex numbers in an Argand diagram Appreciating how arithmetic with complex numbers can be interpreted as geometric transformations Representing equations and inequalities with complex numbers graphically **Assessment:** Work will be submitted regularly to assess understanding Checkpoints are taken at regular intervals, 2 or 3 topics at a time

	Year 13
Autumn Term	**Topics:** **Pure Core** Roots of polynomials; Mathematical Induction; Series; Powers and Roots of Complex Numbers; Complex Numbers and Trigonometry; Lines and Planes in Space; Simultaneous Equations and Planes; Hyperbolic Functions; and Polar Coordinates **Discrete** Graphs and Networks; Algorithms; Network Algorithms; and Decision Making in Project Management **Mechanics (if taken as an option)** Work/Energy/Power; Dimensional Analysis; Momentum and Collisions; Circular Motion; and Centres of Mass **Statistics (if taken as an option)** Counting Principles and Probability; Discrete Random Variables; Poisson Distribution; Non-Parametric Hypothesis Tests; Correlation and Regression; and Chi-Squared Tests **Skills:** **Pure Core** Factorising polynomials and solving equations that may have complex roots Learning a useful link between the roots of a polynomial and its coefficients Using substitutions to solve more complicated equations Using the principle of induction to prove whether patterns continue forever Applying this principle of induction to matrices, number theory, inequalities, sequences, series and differentiation Adapting the method to solve problems in a range of other contexts Using given results for the sums of integers, squares and cubes to find expressions for sums of other series Using a technique called the method of differences to find an expression for the sum of n terms of a series Using the expression for the sum of the first n terms to determine whether an infinite series converges and find its limit Raising complex numbers to integer powers (de Moivre's theorem) Working with complex exponents Finding roots of complex numbers Using roots of unity Finding quadratic factors of polynomials Using a relationship between complex number multiplication and geometric transformations Using de Moivre's theorem to derive trigonometric identities Finding sums of some trigonometric series Finding the equation of a plane in several different forms Finding intersections between lines and planes Calculating angles between lines and planes Calculating the distances between objects in 3-dimensional space Identifying different geometrical configurations of 2 or 3 planes Determining whether a set of simultaneous equations has a unique solution, no solutions or infinitely many solutions Using simultaneous equations to determine the geometrical configuration of 3 planes Defining the hyperbolic functions sinh x, cosh x, tanh x Drawing the graphs of the hyperbolic functions, showing their domain and ranges Writing the inverse hyperbolic functions in terms of logarithms Defining the reciprocal hyperbolic functions sech x, cosech x, coth x Solving equations and prove identities involving hyperbolic functions Differentiating hyperbolic functions Using polar coordinates to represent curves Establishing various properties of curves Converting between polar and cartesian equations of a curve Finding the area enclosed by a polar curve, or between 2 curves **Discrete** Partitioning a set Using the pigeonhole principle

Enumerating arrangements and selections
Using the inclusion-exclusion principle
Learning the terminology of graphs and networks (including isomorphism, planarity, digraph)
Learning about some special types of graph
Using graphs and networks to model problems
Using an algorithmic approach
Tracing through an algorithm
Measuring efficiency and complexity of an algorithm
Interpreting the order of an algorithm
Applying sorting algorithms
Packing items into bins efficiently
Solving problems requiring the least weight path between 2 vertices
Solving problems involving a minimum spanning tree for a network
Solving problems requiring a least weight cycle through every vertex in a network
Finding the least weight route around a network that uses every arc
Constructing and interpreting activity networks
Carrying out critical path analysis
Calculating and interpreting float
Constructing a cascade chart and dealing with scheduling restrictions

Mechanics (if taken as an option)
Calculating the work done by a force, and kinetic energy
Using the work-energy principle
Equating gravitational potential energy to work done against gravity
Performing calculations using power
Understanding the concept of dimensions
Using the language and symbols of dimensional analysis
Understanding the connections between units and dimensions
Checking the validity of a formula by using dimensional considerations
Predicting formulae by using dimensional analysis
Understanding momentum and impulse in mathematical terms with units
Understanding that linear momentum is conserved in a collision between objects that are free to move
Understanding that impulse on a body is equal to the change in momentum
Understanding Newton's experimental law for collisions
Analysing and solving problems involving simple collisions in a straight line
Analysing and solving problems involving simple cases of connected particles
Modelling the motion of a particle moving in a horizontal circular path at a constant speed
Linking linear and angular speed of a particle moving in a horizontal circular path
Finding the acceleration and forces acting on a particle moving in a horizontal circular path
Solving problems relating to motion in a horizontal circular path
Working with a particle moving in a circle with variable speed
Modelling the motion of a particle moving in a circle in a vertical plane
Using the principle of conservation of mechanical energy to solve problems involving a particle moving in a vertical circle
Finding the centre of mass of arrangements of particles, uniform rods and symmetrical uniform laminas
Finding the centres of mass of 2 and 3 dimensional objects of standard shapes
Finding centres of mass of composite bodies, including bent wires
Calculating the work done by a variable force f(x) when displacement is along the x-axis
Understanding and using Hooke's law for elastic strings and springs
Calculating the work done extending an elastic string
Calculating the work done extending or compressing an elastic spring
Including elastic potential energy in problems involving conservation of energy
Using vectors to calculate work done, kinetic energy and power

Statistics (if taken as an option)
Breaking down complicated questions into parts that are easier to count, and then combining them together
Counting the number of ways to permute a set of objects
Counting the number of ways you can choose objects from a group
Applying these tools to problems involving probabilities
Calculating the mean and variance of a discrete random variable
Appreciating how a linear transformation of the variable changes the mean and variance
Using the formulae for expectation and variance of a special distribution called the uniform distribution
Recognising when it is appropriate to use a uniform distribution

	Using the formula for expectation and variance of the binomial distribution Finding probabilities, expectation and variance of the geometric distribution Recognising when it is appropriate to use a geometric distribution Identifying the conditions required for a Poisson distribution to model a situation Using the Poisson formula and calculate Poisson probabilities Calculating the mean, variance and standard deviation of a Poisson variable Using the distribution of the sum of independent Poisson distributions Recognising non-parametric hypothesis tests and select an appropriate test Performing a single-sample sign test and a single-sample Wilcoxon signed-rank test Performing a paired-sample sign test, a Wilcoxon matched-pairs signed-rank test and a Wilcoxon rank-sum test Performing the Wilcoxon signed-rank test and the rank-sum test using a normal approximation Calculating the value of Pearson's product moment correlation coefficient Calculating the value of Spearman's rank correlation coefficient and identify when it is appropriate to use it Conducting hypothesis tests on correlation coefficients Using linear regression to find the equation of a line of best fit Checking if 2 factors are independent Using Yates' correction when required Checking if data comes from a population with proportions Checking if data comes from a known distribution **Assessment:** Work will be submitted regularly to assess understanding Checkpoints will be completed at the end of the AS element of the course Checkpoints are taken at regular intervals depending on the subject area,, 2 or 3 topics at a time
Spring Term	**Topics:** **Pure Core** Further Calculus Techniques; Applications of Calculus; Differential Equations; and Applications of Differential Equations **Discrete** Graphical Linear Programming; The Simplex Algorithm; Game Theory **Mechanics (if taken as an option)** Linear Motion Under a Variable Force; Momentum and Collisions; Circular Motion; and Centres of Mass **Statistics (if taken as an option)** Continuous Distributions; Combining Random Variables; Further Hypothesis Tests; and Confidence Intervals **Skills:** **Pure Core** Differentiating inverse trigonometric and inverse hyperbolic functions Reversing the above results to find integrals of specific forms Using trigonometric and hyperbolic substitutions to find similar integrals Integrating using partial fractions with a quadratic expression in the denominator Finding finite series expansions (called Maclaurin series) of functions Using given results to find the Maclaurin series of more complicated functions Understanding for which values of x these series are valid Finding the value of definite integrals in certain case where a limiting process is required (improper integrals) Finding the volume of a shape by rotating a curve around the x-axis or the y-axis Finding the mean value of a function Understanding and use the language associated with differential equations Solving differential equations given in specific forms Using substitutions to turn differential equations into the required form Using differential equations in modelling, in kinematics and in other contexts Solving the equation for simple harmonic motion and relating the solution to the motion Modelling damped oscillations using second order differential equations and interpreting their solution Solving coupled first order differential equations and using them to model situations with 2 dependent variables

	Discrete Formulating linear programming problems Using slack variables Working with constraints Solving linear programming problems with 2 variables graphically and interpreting solutions Using branch-and-bound for integer programming problems Using a simplex tableau Understanding the meaning of the terms basic and non-basic in the context of solving linear programming problems Understanding iterations of the simplex algorithm geometrically and algebraically Identifying a zero-sum game and understanding how games that are not zero-sum are represented Understanding strict dominance and weak dominance Finding play-safe strategies and understanding what they represent Understanding what a stable solution means Finding optimal mixed strategies for games that do not have stable solutions FInding a Nash equilibrium and what it represents **Mechanics (if taken as an option)** Solving equations of motion of a particle when the velocity is given as function of displacement Solving equations of motion of a particle when the acceleration is a function of velocity or displacement Using connected rates of change to solve linear motion problems Setting up and solving problems which can be modelled as linear motion of a particle acting under a variable force Finding the impulse of a variable force Applying the principles of impulse, conservation of momentum and Newton's experimental law in 2 dimensions using vector notation Calculating the result of oblique impacts Working with the radial and tangential components of the acceleration of a particle moving in a circle Solving problems involving moving particles where only the part of their path is a vertical circle Using integration to find centres of mass of rods of variable density, uniform laminas and uniform solids of revolution Applying knowledge of centres of mass to problems of equilibrium, including suspension of a lamina and toppling or sliding of a lamina acted on by several forces **Statistics (if taken as an option)** Describing probabilities of continuous variables Calculating expected statistics of continuous variables Calculating expected statistics of functions of continuous variables Converting between the probability density function f(x) and the cumulative distribution function Finding the median and quartiles Using probability distributions - the continuous uniform and the exponential Using the cumulative distribution function to find the distribution of the function of a random variable Finding the mean and variance of the sum of two independent random variables Applying these ideas to making predictions about the average or the sum of a sample Learning about the distribution of linear combinations of normal variables Learning about the distribution of the sum or average of many observations from any distribution Being aware of more situations where the normal distribution can be used in hypothesis testing Estimating an interval that a population parameter lies in (confidence interval) **Assessment:** Work will be submitted regularly to assess understanding Checkpoints are taken at regular intervals, 2 or 3 topics at a time January: Mock examinations
Summer Term	The summer term is used to consolidate all the knowledge and skills acquired in the course and prepare students for the final examinations.